Course Title: Engineering Mathematics (3) b Year: 2nd Computer Engineering and Automatic Control.
Course Code: PME2211 Date: / 6 / 2012 (second term) Allowed time: 3 hrs No. of Pages: (2)

Remarks: (Answer the following questions. Assume any missing data...)

Problem number (1) (25 Marks)

(a) Given $A = 0.2/1+0.5/2+0.6/3+1/4+0.7/5+0.3/6+0.1/7$

(i) Calculate the weak $\alpha - cuts$ for $\alpha = 0.4\ ,\ 0.6,\ 0.8$

(ii) Find $\overline{A}$, $CON(A)$, $A \cap \overline{A}$ *and* $A \cup \overline{A}$.

(b) Prove that: If A and B are two fuzzy subsets of the universal set X. Then, the following properties hold for all $\alpha, \beta \in [0,1]$: –

(*i*) $\alpha \leq \beta$ *implies to* $A_\alpha \supseteq A_\beta$

(*ii*) $(A \cap B)_\alpha = A_\alpha \cap B_\alpha$.

(c) Given a fuzzy relations P from A to B defined by :

$$M_P = \begin{bmatrix} 0.4 & 0.8 & 0.5 \\ 0.8 & 0.7 & 0.2 \\ 0.1 & 0.9 & 0.3 \end{bmatrix}$$, and Q from B to C defined by $$M_Q = \begin{bmatrix} 0.1 & 0.8 & 0.2 \\ 0.7 & 0.3 & 0.9 \\ 0.8 & 0.6 & 0.2 \end{bmatrix}$$

Find $M_{P \circ Q}$.

(d)Give two deviations between operations on ordinary sets and fuzzy sets. Illustrate your answer by examples.

Problem number (2) (30 Marks)

(a) If $u(x,y) = x^3 + 6x^2 y - 3x y^2 - 2y^3$ Find an analytic function $f(z) = u(x,y) + i\,v(x,y)$

(b) Solve the equation $\cosh z = 0.2$

(c) Prove that " If $f(z)$ is analytic function in a simply connected domain D and if $f'(z)$ is continuous at each point within and a closed contour C in D, then $\oint_c f(z)\,dz = 0$

(d) Find the general solution of the differential equation:
$x^2 y'' - x y' + (x^2 + 1) y = 0$

Problem number (3) (30 Marks)

(a) Find Taylor and Laurent expansions represent the function f (z) and determine the regions of convergence for each with *center at* $z_0 = 0$ *and* $f(z) = \dfrac{z}{5-z}$

(b) Evaluate the following integrals using complex integration, Gamma and Beta functions:

(*i*) $\displaystyle\oint_{|z|=1} \frac{coz\,2z}{z^4}\,dz$

(*ii*) $\displaystyle\oint_{|z|=3} \frac{e^{2z}\sinh z}{z^2+4}\,dz$

(*iii*) $\displaystyle\oint_{|z-2|=5/2} \frac{z}{\sin z\ (1+\cos z)}\,dz$

(*iv*) $\displaystyle\oint_{|z-2|=1} \frac{\ln z\,\sinh z^2}{z^4\,(z^2+\pi^2)}\,dz$

(*v*) $\displaystyle\oint_{|z|=3} \frac{e^{iz}+\sin z}{(z-\pi)^9\,(z-\pi/2)}\,dz$

(*vi*) $\displaystyle\int_0^{\infty} \frac{\cos 5x}{x^4+16}\,dx$

(*vii*) $\displaystyle\int_0^{\pi/2} (\tan^{3.5}\theta+\tan^{5.5}\theta)\,e^{-\tan^2\theta}\,d\theta$

(*viii*) $\displaystyle\int_0^{2} x^5\sqrt{8-x^3}\,dx$

(*ix*) $\displaystyle\int_0^{\infty} \frac{x^{3/2}}{(1+x^2)^7}\,dx$

Good luck

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